Generalized Kinematics for Deformable Patient-Specific Soft Robots

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I. INTRODUCTION

Cosserat modeling underpins the success of concentric tube robots [1], [2]. Unfortunately, neither concentric tubes nor Cosserat rods apply to fully deformable, stretchable soft actuators. This does not benefit patient-specific soft robots, particularly for under-served 'no option' patients requiring novel deformations (Fig. 1a). We introduce a general kinematic framework to model deformable soft actuators (as Cosserat rods model concentric tube robots). We require that it generalize traditional robot kinematics like Denavit-Hartenberg (D-H) and product of exponentials (POE). We demonstrate this with an application to patient-specific soft robot actuator design.

II. METHODS

Similar to concentric tube robots, this model includes links with the traditional generalized joint angle $q : q_i \leq q \leq q_f \in \Re$ and a new link localization parameter $l : l_s \leq l \leq l_e \in \Re$, both scaled to $[0,1]$. Given desired initial and final shapes as in Fig. 1a with centerlines $c_i, c_f \in \Re^3$ and surfaces $\hat{S}_i, \hat{S}_f \in \Re^3$, our method is as follows.

*1) **Define Link Centerline:*** Blend centerlines as $c(q,l) = (1-q)c_o(l) + qc_f(l)$. This may include intermediate steps. Compute tangent ($\mathbb{T}$), normal ($\mathbb{N}$), and binormal ($\mathbb{B}$) unit vectors using Frenet-Serret formulas along $c(q,l)$.

*2) **Define Link Start-End Frames:*** Attach start frame $\{S\}$ at $c_0(l_s)$ and end frame $\{E\}$ at $c_0(l_e)$ s.t. z-translation from $\{S\}$ is link length (Fig. 1b). Choose final and intermediate $\{E\}$ frames to coincide with $c(q_f, l_e)$ and $c(q, l_e)$.

*3) **Express Link Ends with DH Parameters and POE:*** For DH Parameters, the translation from $\{S\}$ to $\{E\}$ at any $q \in [q_0, q_f]$ can be written as $c(q, l_e)$ and direction cosines provide the relative orientation ${}^sR_e$ of $\{E\}$ w.r.t $\{S\}$.

For POE formulation, decouple transformation from above as ${}^sT_e = e^{\widehat{\xi_1}\theta_1} e^{\widehat{\xi_2}\theta_2}$ where direction $\widehat{\xi_1}$ and magnitude θ_1 refer to prismatic translation, axis $\widehat{\xi_2}$ and magnitude θ_2 to rotation.

*4) **Construct Full Robot Kinematics:*** For each link j, project desired surface points $\hat{S}_j$ onto $c_j(q_j, l_j)$ via $(\mathbb{T}, \mathbb{N}, \mathbb{B})$. This may include a least squares fit and surface spline interpolants with end matching constraints for $S_j(q,l)$. Then,

$$S_{robot}(\bar{q}, \bar{l}) = \sum_{j=1}^{N} S_j(q_j, l_j) + c_{j-1}(q_{j-1}, l_{f,j-1}).$$

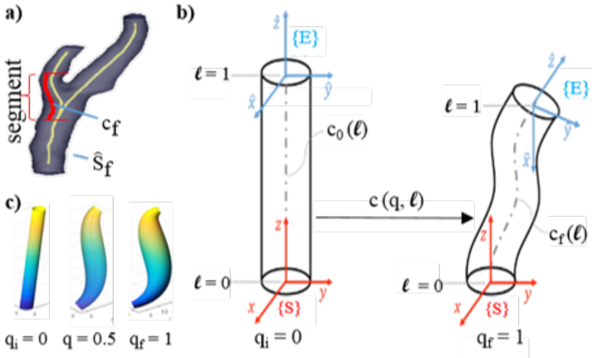

Fig. 1: (a) Sample centerline and surface $\hat{S}_f$ extracted from patient artery scan via VMTK [3]. (b) Labeling convention with initial q_i and final q_f where final link surface S matches $\hat{S}_f$. (c) Results of an axially-symmetric robot segment morphing from generalized joint angle $q_i = 0$ to $q_f = 1$.

III. RESULTS

See Fig. 1. The general DH parameters for each link are:

$${}^sT_e = \begin{bmatrix} \mathbb{N}(q,l) & \mathbb{B}(q,l) & \mathbb{T}(q,l) & c(q,l) \\ 0 & 0 & 0 & 1 \end{bmatrix}_{l=l_e}$$

The resulting general POE formulation results in:
$\theta_1 = \|c(q,l_e) - c(q,l_s)\|$, $\theta_2 = acos[(\mathbb{N}_x + \mathbb{B}_y + \mathbb{T}_z - 1)/2]$, and

$$\widehat{\xi_1} = \begin{bmatrix} \overrightarrow{0_{3\times3}} & \frac{c(q,l_e)-c(q,l_s)}{\|c(q,l_e)-c(q,l_s)\|} \\ \overrightarrow{0_{1\times3}} & 0 \end{bmatrix},$$

$$\widehat{\xi_2} = \frac{1}{2sin(\theta_2)} \begin{bmatrix} 0 & \mathbb{B}_x - \mathbb{N}_y & \mathbb{T}_x - \mathbb{N}_z & 0 \\ \mathbb{N}_y - \mathbb{B}_x & 0 & \mathbb{T}_y - \mathbb{B}_z & 0 \\ \mathbb{N}_z - \mathbb{T}_x & \mathbb{B}_z - \mathbb{T}_y & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ \theta_2 \neq 0.$$

IV. DISCUSSION & CONCLUSION

The results (Fig. 1c) show that our method kinematically describes deformable robot links with stretchable surfaces that match desired shapes. It generalizes the kinematics of soft robots as a superset of both D-H and POE, providing intuitive use for roboticists. Future work will include incorporating realistic stress and strain metrics and automated design of multi-link soft robots from patient-specific data.

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